AN INTRODUCTION TO STRUCTURAL EQUATION MODELING

WITH AN APPLICATION TO THE BLOGOSPHERE

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“Structural equation modeling” or “SEM”

- 1971-1980: 27
- 1981-1990: 118
- 1991-2000: 572
- 2001-2010: 4,348
- 2011-2014: 3,249

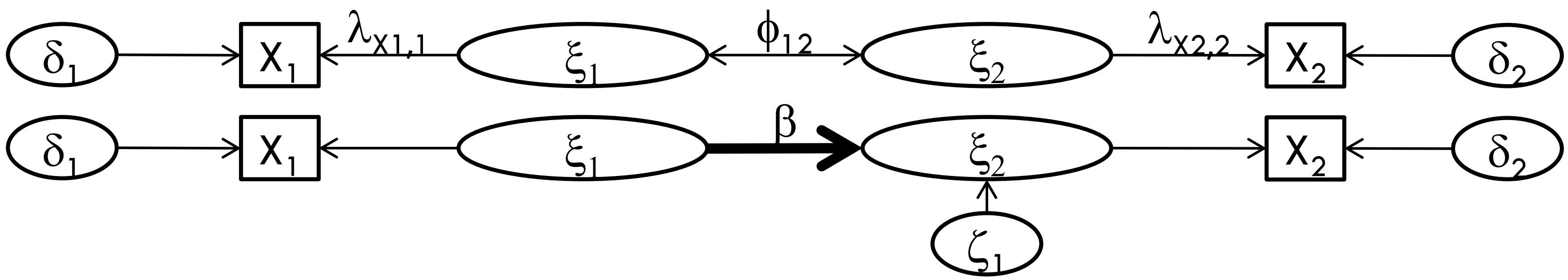
- With its foundation in factor analysis and multiple regression analysis, structural equation modeling is a family of statistical models that seek to explain relationships amongst constructs and between constructs and indicator variables as represented in a measurement model and in a structural model

Structural equation modeling

- Model: A representation of theory that shows how constructs are operationalized by sets of measured variables and how constructs relate to each other
- Measurement model
  - Researcher-specified factor structure concerning the correspondence between measured variables and constructs; goal is to reproduce the observed sample covariance matrix ("**S**") among the indicator variables with an estimated covariance matrix ("$\sum_K$")
- Structural model
  - Based on structural theory; reflects study hypotheses
  - SEM determines whether hypothesized relationships exist between constructs

Measurement model

- Exogenous and endogenous constructs

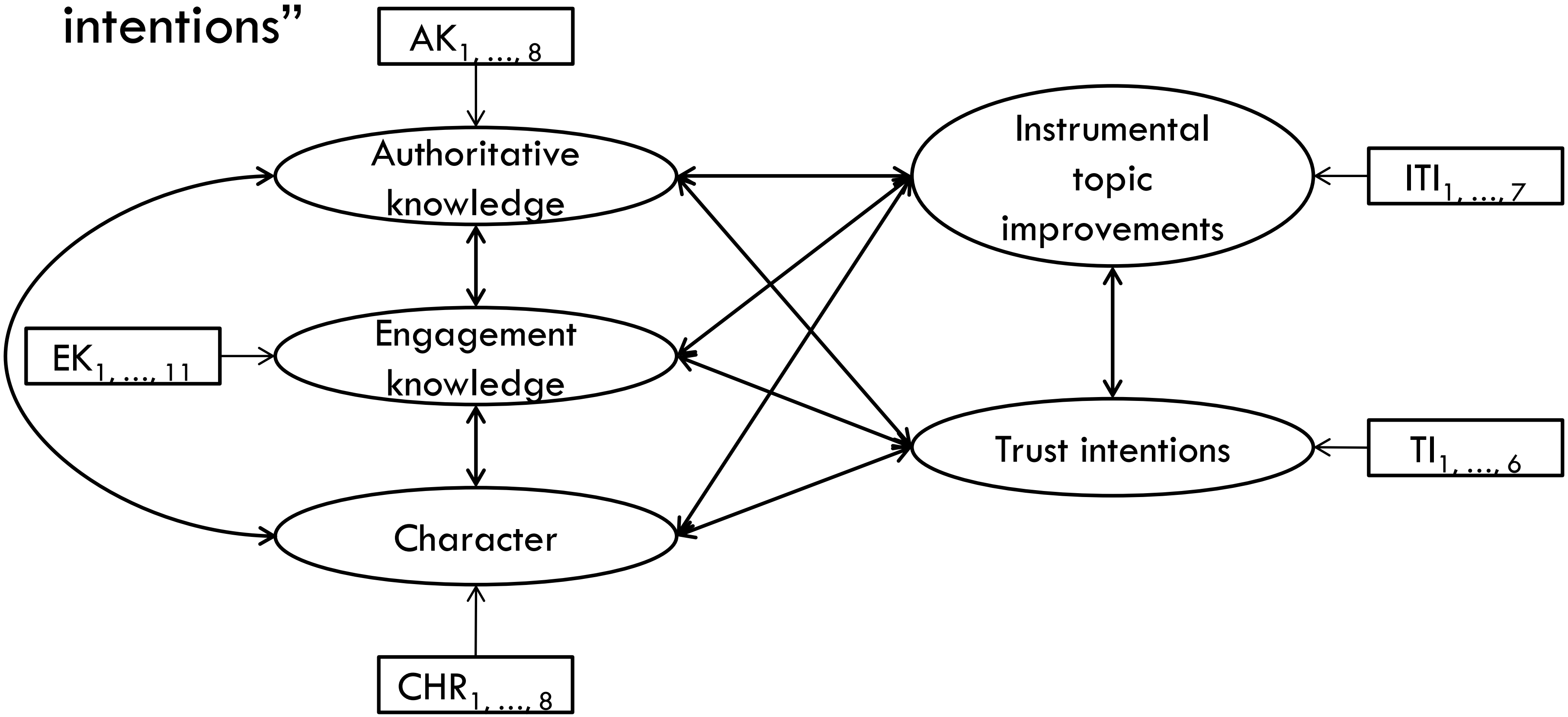


Reflective measurement theory: Assumes the latent constructs cause the measured indicator variables and that error is a result of the inability of the latent constructs to fully explain the indicators.

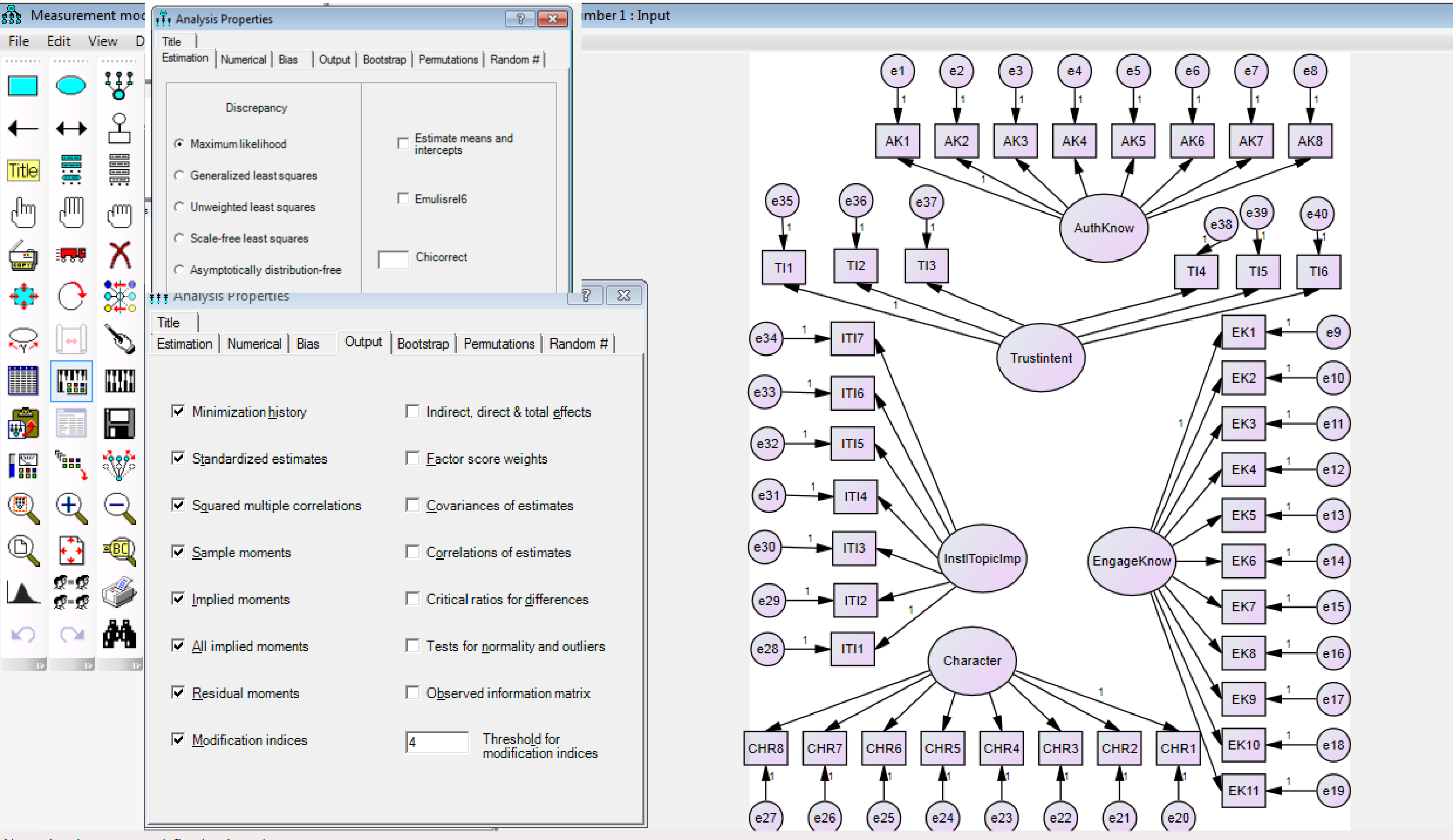
- Canadian blog readers (n = 302)
  - Acceptable sample size, although X^2 sensitive to large sample sizes

My measurement model

- Blogger, blog, and blog reader constructs
  - “Authoritative knowledge;” “engagement knowledge;” “character;” “instrumental topic improvements;” “trust intentions”



Measurement model in AMOS (Without correlations for clarity)



Measurement model considerations

- Goodness of fit: Multiple tests are best
  - X^2 or X^2/df
    - Null hypothesis is no difference between the two covariance matrices; want insignificant X^2 but can expect $p < .05$ with large samples and complex measurement models
  - Absolute (e.g., GFI, RMSEA) and incremental (e.g., CFI) indices
    - Unlike absolute fit indices, incremental fit indices compare to a null model in which all measured variables are specified as uncorrelated
  - Goodness-of-fit indices (e.g., comparative fit index)
    - Guideline: CFI $\geq .90$
  - Badness-of-fit indices (e.g., root mean square error of approximation)
    - Guideline: RMSEA $\leq .10$
- Construct validity: Face, convergent, discriminant, and nomological
- Construct reliability

CFA Results

- Sample of Canadian blog readers (n = 302)
  - X^2 =2,408.44; df = 730; p = .000
- X^2 is significant, indicating that the observed covariance matrix does not match the estimated covariance matrix within sampling variance.
  - Significant X^2 is common.
- Other fit measures
  - CFI = .77
  - RMSEA = .09
- Things to check:
  - Loadings (significance; ≥ .7 or .5)
  - Standardized residuals (|4|)
  - Modification indices, although the sole goal is not model fit
    - Requires no missing data

Actions taken and revised CFA results

- Action
  - All λs significant but two variables removed (standardized loadings < .5); loadings < .7 are a judgment call
    - (Standardized) regression weights in AMOS
  - Inspections of standardized residuals resulted in removal of several variables
    - Check standardized residuals > $|4.0|$ or $|2.5|$
- Revised CFA results
  - $X^2 = 666.28$; df = 242; $p = .000$
  - CFI = .90
  - RMSEA = .07

Construct validity

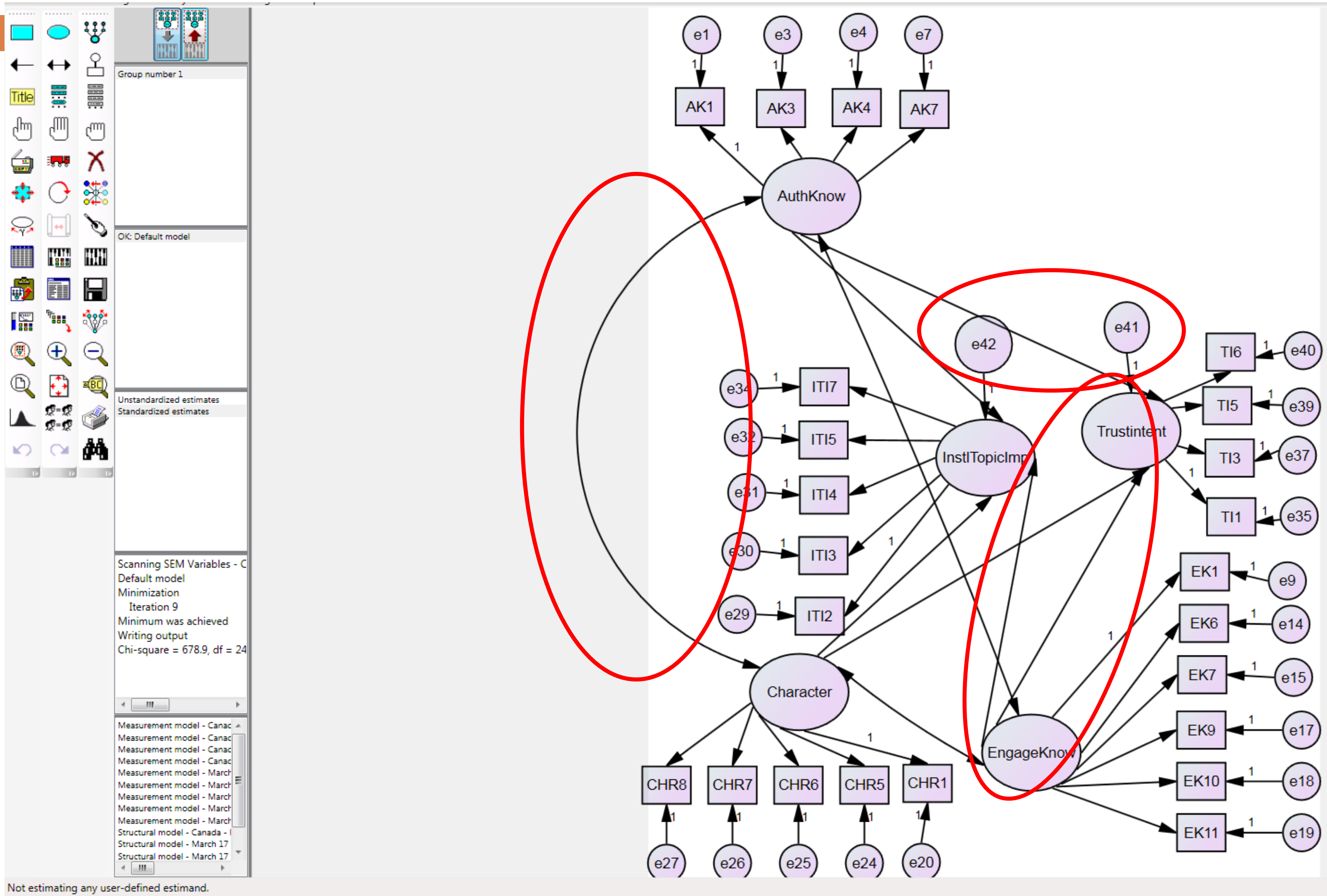
- Face validity: Item content is consistent with the construct's definition
- Convergent validity
  - Factor loadings (ideally .7 or higher) and average variance extracted (should be .5 or higher)
    - AK: .58; EK: .56; CHR: .52; ITI: .68; TI: .56
- Discriminant validity
  - Check interconstruct variance
    - Compare the variance-extracted estimates for each factor with the squared interconstruct correlations associated with that factor
      - Average variance extracted should be greater than .5
      - No squared interconstruct correlation > .5
  - Also specifying $r_{AK,EK} = 1$ did not improve model fit
- Nomological validity: Check correlations for sense and constructs' relationships to non-model variables
  - E.g., $r_{AK,ITI} > r_{AK,TI}$ (.55 versus .26)

Construct reliability

- Reliability is a measure of the internal consistency of the observed indicator variables
  - Measures
    - Cronbach alpha (SPSS)
    - Composite reliability (Need to calculate)
  - Reliability should be .7 or higher to indicate adequate convergence or internal consistency

Construct	Cronbach alpha	Composite reliability
Authoritative know.	.84	.84
Engagement know.	.88	.88
Character	.84	.85
Instrumental topic imp.	.90	.91
Trust intentions	.83	.83

Structural model in AMOS



Structural model analysis results

Amos Output

Structural model - Canada - Readers only - March 19 2014 - With double arrows - After variable removal.amw

- Analysis Summary
- Notes for Group
- Variable Summary
- Parameter Summary
- Sample Moments
- Notes for Model
- Estimates
  - Scalars
    - Regression Weights:
    - Standardized Regression Weights:
    - Covariances:
    - Correlations:
    - Variances:
    - Squared Multiple Correlations:
  - Matrices
- Modification Indices
- Minimization History
- Model Fit
- Execution Time

The probability of getting a critical ratio as large as 1.324 in absolute value is .186. In other words, the regression weight for **AuthKnow** in the prediction of **Trustintent** is not significantly different from zero at the 0.05 level (two-tailed).

These statements are approximately correct for large samples under suitable assumptions. (See Assumptions.)

Standardized Regression Weights: (Group number 1 - Default model)

			Estimate
InstlTopicImp	<---	AuthKnow	.223
InstlTopicImp	<---	EngageKnow	.327
InstlTopicImp	<---	Character	.323
Trustintent	<---	AuthKnow	-.107
Trustintent	<---	EngageKnow	.138
Trustintent	<---	Character	.308
Trustintent	<---	InstlTopicImp	.305

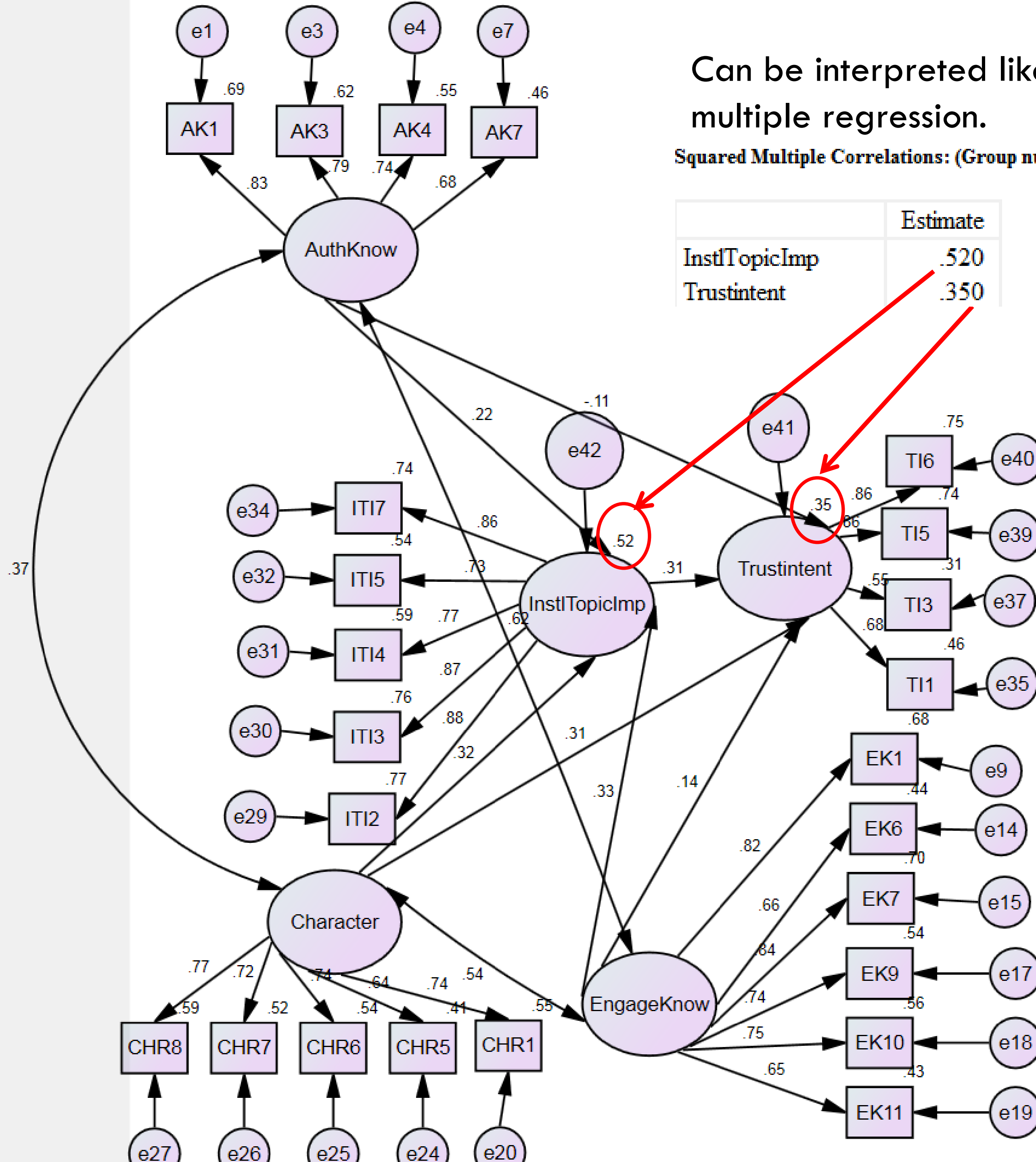
Regression Weights: (Group number 1 - Default model)

			Estimate	S.E.	C.R.	P
InstlTopicImp	<---	AuthKnow	.204	.062	3.300	***
InstlTopicImp	<---	EngageKnow	.360	.083	4.314	***
InstlTopicImp	<---	Character	.349	.069	5.057	***
Trustintent	<---	AuthKnow	-.072	.055	-1.324	.186
Trustintent	<---	EngageKnow	.112	.074	1.512	.131
Trustintent	<---	Character	.247	.065	3.788	***
Trustintent	<---	InstlTopicImp	.226	.065	3.502	***
AK1	<---	AuthKnow	1.000			
AK7	<---	AuthKnow	.854	.071	12.035	***
EK1	<---	EngageKnow	1.000			
EK6	<---	EngageKnow	.924	.075	12.277	***
EK7	<---	EngageKnow	1.000	.060	16.647	***
EK9	<---	EngageKnow	.998	.071	14.016	***
EK10	<---	EngageKnow	.950	.066	14.352	***
EK11	<---	EngageKnow	.830	.069	12.013	***
CHR5	<---	Character	.879	.084	10.414	***
CHR6	<---	Character	1.093	.091	11.976	***
CHR7	<---	Character	.826	.070	11.748	***
CHR8	<---	Character	1.037	.083	12.478	***
ITI2	<---	InstlTopicImp	1.000			
ITI3	<---	InstlTopicImp	.897	.043	20.661	***
ITI4	<---	InstlTopicImp	.822	.050	16.579	***
ITI5	<---	InstlTopicImp	.896	.058	15.373	***
ITI7	<---	InstlTopicImp	.951	.047	20.208	***
AK3	<---	AuthKnow	.969	.068	14.349	***
AK4	<---	AuthKnow	.891	.067	13.338	***
CHR1	<---	Character	1.000			
TI3	<---	Trustintent	.760	.088	8.685	***
TI1	<---	Trustintent	1.000			
TI5	<---	Trustintent	1.223	.096	12.678	***
TI6	<---	Trustintent	1.274	.100	12.696	***

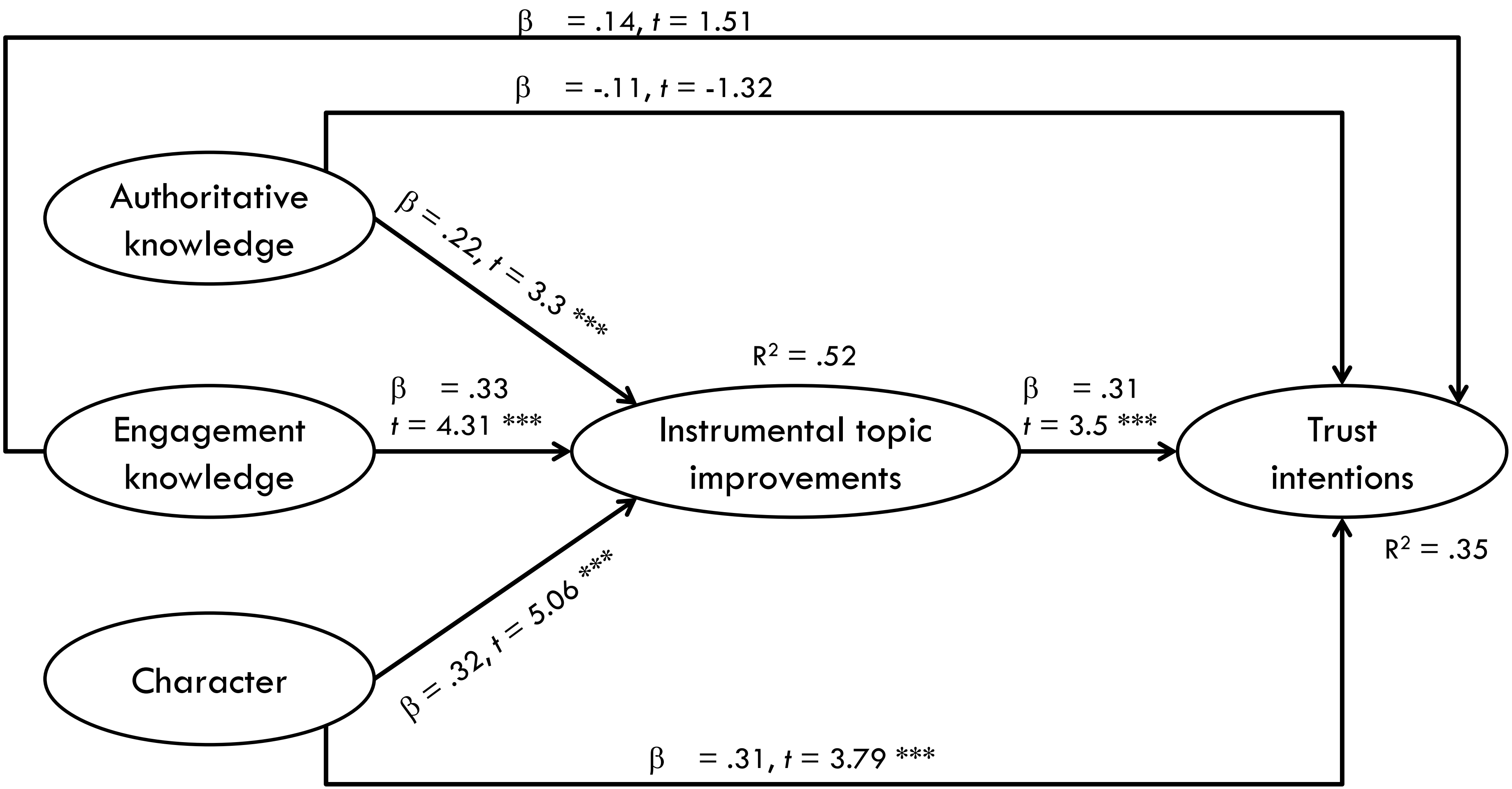
Can be interpreted like the R^2 in multiple regression.

Squared Multiple Correlations: (Group number 1 - Default model)

	Estimate
InstlTopicImp	.520
Trustintent	.350



In another graphical form



*** $p < .001$

Canadian versus Chinese blog readers

- First, translational equivalence
  - Translation-back translation
- Then, metric invariance
  - Ensures that the measures have the same meaning and are used in the same way by different groups of respondents
- Next, scalar invariance
  - Ensure that amounts (e.g., means) have the same meaning among by different groups of respondents